WEEKLY TEST OYJ TEST - 24 R & B
SOLUTION Date 06-10-2019

[PHYSICS]

1.

According to lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For given concave lens,

$$R_1 = -3 \text{ cm} \quad \text{and} \quad R_2 = -4 \text{ cm}$$

$$\therefore \qquad \frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{-3} + \frac{1}{4}\right)$$

$$\text{or} \qquad \frac{1}{v} - \frac{1}{(-12)} = (1.5 - 1)\left(\frac{-4 + 3}{12}\right)$$

$$\text{or} \qquad \frac{1}{v} + \frac{1}{12} = 0.5 \times \frac{-1}{12} = \frac{-1}{24}$$

$$\text{or} \qquad \frac{1}{v} = -\frac{1}{24} - \frac{1}{12} = \frac{-1-2}{24} = -\frac{1}{8}$$

$$\text{or} \qquad v = -8 \text{ cm.}$$

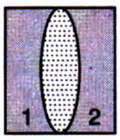
2.

As shown in the figure, the system is equivalent to combination of three thin lens in contact.

$$\therefore \qquad \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

By lens maker's formula,

$$\frac{1}{f_1} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{\infty} - \frac{1}{25}\right) = \frac{-1}{50}$$



$$\frac{1}{f_2} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{25} + \frac{1}{20}\right) = \frac{3}{100}$$

$$\frac{1}{f_3} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{-20} - \frac{1}{\infty}\right) = -\frac{1}{40}$$

$$\frac{1}{f} = \frac{1}{5}\left[-\frac{1}{10} + \frac{3}{20} - \frac{1}{8}\right]$$

$$= \frac{1}{5}\left[\frac{-8 + 12 - 10}{80}\right] = \frac{1}{5}\left[\frac{-6}{80}\right]$$

$$\text{or} \qquad f = -\frac{400}{6} \text{ cm} = -66.6 \text{ cm}$$

Hence, the system behaves as a concave lens of focal length 66.6 cm.

3.

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \quad ...(i)$$

Given, $f = 10$ cm (as lens is converging)

$u = -8$ cm (as object is placed on left side of the lens)

Substituting these values in eqn. (i), we get;

$$\frac{1}{10}=\frac{1}{v}-\frac{1}{-8} \quad \text{or} \quad \frac{1}{v}=\frac{1}{10}-\frac{1}{8}$$

or $$\frac{1}{v}=\frac{8-10}{80}$$

or $$v=\frac{80}{-2}=-40 \text{ cm}$$

Hence, magnification produced by the lens,

$$m=\frac{v}{u}=\frac{-40}{-8}=5.$$

4.

Here, $f_1 = 20$ cm, $f_2 = 25$ cm.

The effective power of the combination is,

$$P=P_1+P_2=\frac{1}{f_1}+\frac{1}{f_2}$$

$$=\frac{100}{20}+\frac{100}{25} \quad \left(\because P \text{ (in dioptre)}=\frac{100}{f \text{ (in cm)}}\right)$$

$$= 5D + 4D = 9D.$$

5.

Focal length of convex lens, $f_1 = 25$ cm

Focal length of concave lens, $f_2 = -25$ cm

Power of combination in dioptres,

$$P=P_1+P_2=\frac{100}{f_1}+\frac{100}{f_2}=\frac{100}{25}-\frac{100}{25}=0.$$

6.

7.

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$$

or $$\frac{1}{f}=\frac{1}{0.2}+\frac{1}{0.2}-\frac{0.5}{(0.2)(0.2)}$$

or $$1/f=5+5-0.5\times5\times5$$

or $$1/f=10-12.5=-2.5$$

or $$f=-(1/2.5)=-0.4 \text{ m.}$$

8.

$$R=-24 \text{ cm}, f=-12 \text{ cm}, m=1.5$$

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}; \quad \frac{1}{1.5u}+\frac{1}{u}=-\frac{1}{12}$$

$$\frac{2.5}{1.5u}=-\frac{1}{12} \quad \text{or} \quad u=-20 \text{ cm}$$

–ve sign shows that object is placed infront of a convex mirror.

9.

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \quad \text{or} \quad \frac{1}{-20}+\frac{1}{v}=\frac{1}{20}$$

$\therefore \quad v = 10$ cm

$$m=\frac{v}{u}=\frac{h_2}{h_1}$$

or $\quad \frac{10}{20}=\frac{h_2}{2\text{ mm}}$ or $h_2 = 1$ mm.

10.

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

$$=(1.5-1)\left(\frac{1}{10}+\frac{1}{10}\right) = 0.1$$

$f = 10$ cm

$\therefore \quad R = 2f = 2\times 10 = 20$ cm.

11.

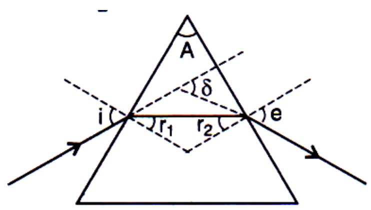
$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$v = 3u$, $u = 20$ cm and both v and u are –ve.

12.

We know that when convex lens is made of three different materials, then it has three refractive indices and therefore three focal lengths. Hence, number of images formed by the lens will be three.

13.



Here, $\quad i = 60°,\ A = 30°,\ \delta = 30°$

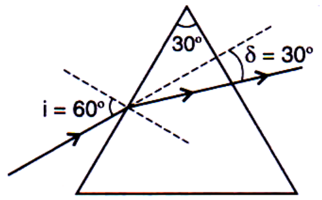
As $\quad i + e = A + \delta$

$$e = A + \delta - i = 30°+30°-60° = 0°$$

Hence emergent ray is normal to the surface.

$$e = 0° \text{ or } r_2 = 0°$$

As $\quad r_1 + r_2 = A$



$\therefore \quad r_1 = A - r_2 = 30°-0° = 30°$

$$\mu=\frac{\sin i}{\sin r_1}=\frac{\sin 60°}{\sin 30°}=\frac{\sqrt{3}}{2}\times\frac{2}{1}=\sqrt{3}=1.732$$

14.

For dispersion without deviation

$$\delta_1 + \delta_2 = 0$$

$$(\mu_1 - 1)A_1 + (\mu_2 - 1)A_2 = 0$$

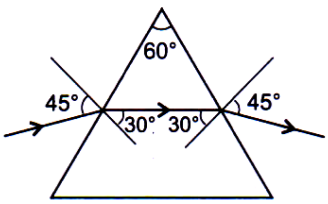
$$A_2 = -\frac{(\mu_1 - 1)A_1}{(\mu_2 - 1)}$$

Substituting the given values, we get

$$A_2 = -\frac{(1.5 - 1)15°}{(1.75 - 1)} = -10°$$

–ve sign shows that prism must be joined in opposition.

15.



As ray pass symmetrically through prism, hence

$$\delta_{min} = (i + e) - A = 30°$$

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}} = \sqrt{2}$$

[CHEMISTRY]

16.

The given formula $CoCl_3 \cdot 6NH_3$ confirms the (b) answer only. Moreover,

$$\text{Moles of complex} = \frac{2.675}{267.5} = 0.01$$

$$\text{Moles of AgCl} = \frac{4.78}{143.5} = 0.033$$

This shows $3Cl^-$ ions in ionic sphere.

17.

$[Cu(H_2O)_4]^{2+}$ has weak ligand H_2O and absorbs red light of visible spectrum. The colour that appears is blue. $[Cu(H_2O)_4]^{2+}$ is labile complex and changes to $[Cu(NH_3)_4]^{2+}$ by the addition of strong ligand NH_3. It has strong ligand NH_3 and absorbs yellow light of visible spectrum. The colour that appears is deep blue (indigo).

18.

Both show *cis* and *trans*-geometrical isomerism. *Cis*-isomer appears in *d* and *l* optical isomers. Hence, both have total 3 isomers each.

19.

Neutral cases of carbonyls slightly increase the bond length of C — O bond by donation in antibonding molecular orbitals of CO molecule.

20.
21.

22.

This compound is *cis*-platin anticancer compound.

In $[\mathbf{Co(en)_3}]^{3+}$ has all (en) in *cis*- positions and hence, shows optical isomerism.

23.

The structure shows absence of any ion in ionic sphere.

24.

It is a chelate of five members.

25.

'Low spin' means pairing of electrons takes place. d^4 | ↑ | ↑ | ↑ | ↑ | | changes to d^4 | ↑↓ | ↑ | ↑ | | | with two vacant orbitals. This will give d^2sp^3 octahedral system.

26.

$[Cr(NH_3)_6]Cl_3$ gives four ions in water.

27.
28.

Because $\frac{1}{3}$ of $3Cl^-$ are precipitated as AgCl, only one Cl^- is present in ionic sphere.

29.
30.

$[CrCl_2(H_2O)_4]Cl$

Moles of Cl^- = Moles of AgCl = $MV = 0.01 \times \frac{100}{1000} = \mathbf{0.001}$

[MATHEMATICS]

31.

Resultant of $\vec{a}$ and $\vec{b}$ means $\vec{a}+\vec{b}$.

Here $\vec{a}+\vec{b} = (2\hat{i}+\hat{j}+\hat{k})+(\hat{i}+2\hat{j}+3\hat{k})$

$= 3\hat{i}+3\hat{j}+4\hat{k}$

$\therefore |\vec{a}+\vec{b}| = \sqrt{3^2+3^2+4^2} = \sqrt{9+9+16}$

$= \sqrt{34}$.

32.

$\overrightarrow{AE} = \overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DE}$

$= \overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}-\overrightarrow{ED}$

$= \vec{a}+\vec{b}+\vec{c}-\overrightarrow{AB} = \vec{a}+\vec{b}+\vec{c}-\vec{a}$.

33.

If P.V. of the fourth vertex be $\vec{v}$, then

$$\frac{\vec{v}+(\hat{i}+3\hat{j}+5\hat{k})}{2} = \frac{(\hat{i}+\hat{j}+\hat{k})+(7\hat{i}+9\hat{j}+11\hat{k})}{2}$$

($\because$ diagonals bisect each other)

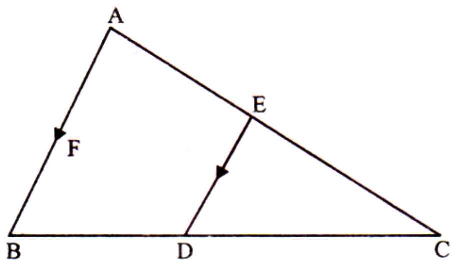
34.

P.V. of P $= \dfrac{3\vec{b}+\vec{a}}{3+1}$ and

P.V. of Q $= \dfrac{\text{P.V. of A} + \text{P.V. of P}}{2}$

35.

From geometry, we know that [ED] is equal to half of [AB] in length and is parallel to [AB], therefore, $\overrightarrow{AB} = 2\,\overrightarrow{ED}$.



36.

Taking I, the incentre of the triangle as origin, we have

$$\frac{|BC|\,\vec{IA}+|CA|\,\vec{IB}+|AB|\,\vec{IC}}{|BC|+|CA|+|AB|}$$

= P.V. of the incentre

(Using incentre formula)

$$\frac{|BC|\,\vec{IA}+|CA|\,\vec{IB}+|AB|\,\vec{IC}}{|BC|+|CA|+|AB|} = \text{P.V. of I} = \vec{0}$$

$$\Rightarrow |BC|\,\vec{IA}+|CA|\,\vec{IB}+|AB|\,\vec{IC} = \vec{0}$$

37.

Here, $\vec{a}+\vec{b}=\alpha\vec{c}$ and $\vec{b}+\vec{c}=\beta\vec{a}$ ...(1)

where α and β are scalars.

Eliminating $\vec{c}$ between these two equations, we get

$\vec{a}+\vec{b}=\alpha(\beta\vec{a}-\vec{b})$

$\Rightarrow (1-\alpha\beta)\vec{a}+(1+\alpha)\vec{b}=0$

$\Rightarrow 1-\alpha\beta=0$ and $1+\alpha=0$

($\because$ $\vec{a}$ and $\vec{b}$ are non-collinear)

$\Rightarrow \alpha=-1, \beta=-1.$

Hence, from (1), $\vec{a}+\vec{b}=(-1)\vec{c}$

$\Rightarrow \vec{a}+\vec{b}+\vec{c}=0.$

38.

$\overrightarrow{AC}-\overrightarrow{BD}=(\overrightarrow{AB}+\overrightarrow{BC})-(\overrightarrow{BC}+\overrightarrow{CD})$

$=\overrightarrow{AB}-\overrightarrow{CD}=\overrightarrow{AB}+\overrightarrow{DC}$

$=\overrightarrow{AB}+\overrightarrow{AB}=2\overrightarrow{AB}$

39.

Let D be the mid-point of segment [BC], then

$2\overrightarrow{AD}=\overrightarrow{AB}+\overrightarrow{AC}$

$\Rightarrow \overrightarrow{AD}=\frac{1}{2}\left\{(3\hat{i}+4\hat{k})+(5\hat{i}-2\hat{j}+4\hat{k})\right\}$

$=4\hat{i}-\hat{j}+4\hat{k}$

Hence required length of the median

$=|AD|=\overrightarrow{AD}$

$=\sqrt{4^2+(-1)^2+4^2}=\sqrt{33}\cdot$

40.

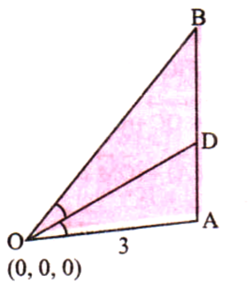


FIGURE 13.26

Here, $|OA|=|\overrightarrow{OA}|=|$P.V. of A$|$

$=|2\hat{i}+2\hat{j}+\hat{k}|$

$=\sqrt{2^2+2^2+1^2}=3$

$|OB|=|\overrightarrow{OB}|=|$P.V. of B$|$

$=|2\hat{i}+4\hat{j}+4\hat{k}|$

$$= \sqrt{2^2 + 4^2 + 4^2} = 6$$

Let internal bisector of angle $\angle BOA$ meet [AB] in D, then D divides [AB] in the ratio

$| OA | : | OB |$

i.e., 3 : 6 or 1 : 2

Hence, P.V. of D

$$= \frac{1 \times \text{P.V. of B} + 2 \times \text{P.V. of A}}{1+2}$$

$$= \frac{1(2\hat{i} + 4\hat{j} + 4\hat{k}) + 2(2\hat{i} + 2\hat{j} + \hat{k})}{3}$$

$$= 2\hat{i} + \frac{8}{3}\hat{j} + 2\hat{k}$$

Hence, $| OD | = | \overrightarrow{OD} | = |$ P.V. of D $|$

$$= \sqrt{2^2 + \left(\frac{8}{3}\right)^2 + 2^2}$$

$$= \sqrt{4 + 4 + \frac{64}{9}} = \frac{\sqrt{136}}{3}.$$

41.

$$\left(\frac{1}{8}\hat{i} - \frac{3}{8}\hat{j} + \frac{1}{4}\hat{k}\right) \cdot (2\hat{i} + 4\hat{j} + 5\hat{k})$$

$$= \frac{2}{8} - \frac{12}{8} + \frac{5}{4} = 0.$$

42.

$$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} \Rightarrow \vec{a} \times \vec{b} - \vec{b} \times \vec{c} = \vec{0}$$

$$\Rightarrow \vec{a} \times \vec{b} + \vec{c} \times \vec{b} = \vec{0} \Rightarrow (\vec{a} + \vec{c}) \times \vec{b} = \vec{0}$$

$\Rightarrow$ either $\vec{a} + \vec{c} = \vec{0}$ or $\vec{a} + \vec{c}$ is parallel to

43.

$$| \vec{a} + \vec{b} | < 1 \Rightarrow | \vec{a} + \vec{b} |^2 < 1$$

$$\Rightarrow (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) < 1$$

$$\Rightarrow | \vec{a} |^2 + | \vec{b} |^2 + 2| \vec{a} || \vec{b} | \cos\theta < 1$$

$$\Rightarrow 1 + 1 + 2\cos\theta < 1$$

$$\Rightarrow 2\cos\theta < -1 \Rightarrow \cos\theta < -\frac{1}{2}$$

$$\Rightarrow -1 \le \cos\theta < -\frac{1}{2}$$

$(\because -1 \le \cos\theta \le 1$ for all $\theta)$

$$\Rightarrow \cos\pi \le \cos\theta < \cos\frac{2\pi}{3} \Rightarrow \pi \ge \theta > \frac{2\pi}{3}.$$

44.

$$\frac{(\hat{i}-2\hat{j}+\hat{k})\cdot(4\hat{i}-4\hat{j}+7\hat{k})}{\sqrt{4^2+(-4)^2+7^2}}$$

$$=\frac{4+8+7}{\sqrt{81}}=\frac{19}{9}.$$

45.

$$\vec{\alpha}\times\vec{\beta}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & -1\\ -1 & 2 & -4\end{vmatrix}=-10\hat{i}+9\hat{j}+7\hat{k}$$

and $$\vec{\alpha}\times\vec{\gamma}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & -1\\ 1 & 1 & 1\end{vmatrix}=4\hat{i}-3\hat{j}-\hat{k}.$$